RA 13 (There was no RA 12)

8.4 Mathematical Induction

- Induction : a proof technique that is useful for proving statements about elements in a sequence.

- Two components of an inductive proof:
  - Base case establishes that the theorem is true for the first value in a sequence.
  - Inductive step establishes that if the theorem is true for k, then the theorem also holds for $k+1$.

- The principal of mathematical induction states that if the base case (for $n=1$) is true and the inductive step is true, then the theorem holds for all positive integers.

- In the statement "$S(K)$ implies $S(K+1)$" of the inductive step, the supposition that $S(K)$ is true is called the inductive hypothesis.

8.5 More Inductive Proofs

- This section had more examples of inductive proofs.

8.6 Strong Induction and well ordering

- The principle of strong induction assumes that the fact to be proven holds for all values less than or equal to k and proves that the fact holds for K+1.

Inductive Step for weak induction

For all $k \geq 1$, $S(K) \rightarrow S(K+1)$

$k=1$: $S(1) \rightarrow S(2)$

$k=2$: $S(2) \rightarrow S(3)$

$k=3$: $S(3) \rightarrow S(4)$

$\vdots \qquad \ddots$

Inductive Step for strong induction:

For all $K \geq 1$, $[S(0) \wedge S(1) \ldots \wedge S(K)] \rightarrow S(K+1)$

$k=1$: $[S(0) \wedge S(1)] \rightarrow S(2)$

$k=2$: $[S(0) \wedge S(1) \wedge S(2)] \rightarrow S(3)$

$k=3$: $[S(0) \wedge S(1) \wedge S(2) \wedge S(3)] \rightarrow S(4)$

$\vdots \qquad \ddots$

Generalized strong induction: multiple base cases

- The base case for proof by strong induction establishes that $S(n)$ holds for $n=a$ through b, where a and b are constants.
- The inductive step in a proof by strong induction assumes $S(j)$ is true for all values of j in the range from a through some integer $k \geq b$ and then proves that theorem holds for $K+1$.

- The well-ordering principle says that any nonempty subset of nonnegative integers has a smallest element.